Groups 2023 - Northern Group Theory conference in honour of Bernd Fischer

Thursday		**Friday**		**Saturday**	
8.30	Registration				
9.00 – 9.45	Roney-Dougal	9.00 – 9.45	Malle	9.30 – 10.15	van der Kallen
10.00 – 10.45	Camina	10.00 – 10.45	Grazian	10.15 – 10.45	Tea/Coffee
10.45 – 11.15	Tea/Coffee	10.45 – 11.15	Tea/Coffee	10.45 – 11.30	Maglione
11.15 – 12.00	Benson	11.15 – 12.00	Piterman	11.45 – 12.30	Burness
12.00 – 13.30	Lunch	12.00 – 13.00	Lunch	12.30	Lunch
13.30 – 14.30	Poster (Coffee)	13.00 – 14.00	Tea/Coffee		
14.30 – 15.15	Weiss	14.00 – 14.15	Welcome (Ringel)		
15.20 – 16.05	Rees	14.15 – 15.00	Capdeboscq		
16.05 – 16.35	Tea/Coffee	15.05 – 15.50	Cuypers		
16.35 – 17.20	Gobet	15.50 – 16.30	Tea/Coffee		
17.25 – 18.10	Witzel	16.30 – 17.15	Röhrle		
		17.15	Gina Friesicke (Violin)		
		19.00	Dinner		

Dave Benson
Title: *Finite groups, cohomology, fusion, singularities, and the nucleus*
Abstract: Cohomology of finite groups is a meeting point for algebra and topology. It has a strong bearing on the structure of the stable module category, and reflects the fusion of subgroups of a Sylow subgroup. We discuss a circle of ideas around cohomology, classifying spaces, p-completion, fusion systems, singularity categories, and the concept of the nucleus. The nucleus is defined in terms of p-subgroup centralisers that are not p-nilpotent, and is intimately connected with the singularities of the classifying space at the prime p.

Tim Burness
Title: *Simple groups, Sylow subgroups and generation*
Abstract: By a theorem of Aschbacher and Guralnick, every finite group can be generated by a pair of conjugate soluble subgroups. The proof uses CFSG and it relies on the fact that every finite simple group can be generated by two Sylow p-subgroups for some prime p. The latter result has been extended in recent work by Breuer and Guralnick, who conjecture that if G is simple and r,s are any prime divisors of |G|, then G is generated by a Sylow r-subgroup and a Sylow s-subgroup. In this talk, I will report on progress towards a proof of this conjecture, which relies on a probabilistic approach. This is joint work with Bob Guralnick.

Rachel Camina
Title: *Coverings of groups*
Abstract: Suppose G is a group, a covering of G is a set of proper subgroups whose union is G. The study of coverings of groups has a long history. In 2017 I was asked a question about coverings of finite p-groups. We now have an answer. I will talk about how we reached this answer and how we were led to the infinite world of pro-p groups. This is joint work with Yiftach Barnea, Mariagrazia Bianchi, Mikhail Ershov, Mark L Lewis and Emanuele Pacifici.

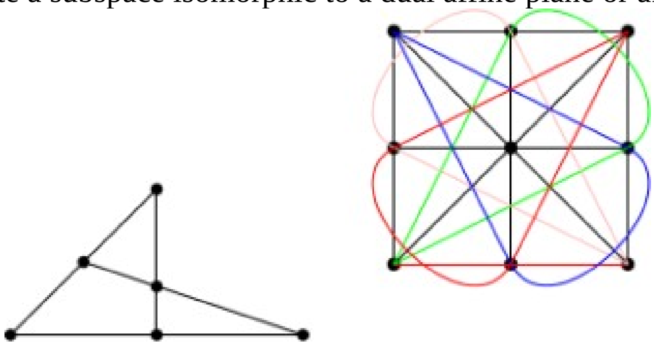
Inna Capdeboscq
Title: *Simple groups of even and 3-type*
Abstract: In this talk we will discuss the classification of finite simple groups that are of simultaneously even- and p-types. In particular, the sporadic groups Fi_{22}, Fi_{23} and Fi_{24} appear here as the groups of even- and 3-types.
This is a joint work with R. Lyons and R.Solomon.

Hans Cuypers

Title: *Transposition Lie Algebras*

Abstract: Associated to a class of 3-transpositions D in a group $\langle G = D\rangle$ there is associated the corresponding Fischer space, i.e. a partial linear space with point set D and as lines the triples $\{d,e,d^e=e^d\}$, where d and e are two noncommuting elements from D.

Fischer spaces can be characterized as partial linear spaces with 3 points per line in which any two intersecting lines generate a subspace isomorphic to a dual affine plane or affine plane.



Given a Fischer space, and point d the map τ_d fixing d, as well as all the points not collinear with d, but on each line through d switching the points different from d is an automorphism of the Fischer space. The set of all such maps forms a normal set of 3-transpositions inside the automorphism group of the Fischer space. Fischer spaces form an important tool in the study of 3-transposition groups.

Within this talk we will show how we can use partial linear spaces of order 3 to study Lie algebras. In particular, we show that several classical Lie algebras can be obtained from Fischer spaces with only dual affine planes and use this to provide presentations of these algebras.

Thomas Gobet

Title: *On some quotients of braid groups and Garside groups*

Abstract: The symmetric group on n letters can be obtained as a quotient of the n-stranded braid group by adding the relations $x^2=1$, where x is any standard generator of the braid group. The quotients by the relations $x^k=1$, where $k \geq 3$ are much less understood, even if some of them appear to be complex reflection groups. The same question can be asked for Garside groups, which are generalizations of braid groups. That is, given any Garside group with a homogeneous presentation: can one understand the quotient of such a group by the relations $x^k=1$, where $k \geq 2$ and x is an atom?

We give a positive answer to this question for torus knot groups, which form an infinite family of Garside groups generalizing the 3-stranded braid group, and arbitrary parameter $k \geq 2$. We obtain an infinite family of groups, and show that a group is a finite group in this family if and only if it is a complex reflection group of rank 2 with a single orbit of reflecting hyperplanes. This is achieved by showing that, even if the quotient is infinite, it behaves like a complex reflection group in some sense. We thus call these quotients "toric reflection groups". We also show that toric reflection groups have a cyclic center, and that the quotient of a toric reflection group by its center can naturally be identified with the rotation subgroup of a Coxeter group of rank 3. This allows one to classify toric reflection groups and to deduce that, even when a toric reflection group is infinite, the corresponding torus knot group behaves like its "braid group", while when the group is finite we recover its complex braid group.

Valentina Grazian

Title: *On the lookout for exotic fusion systems*

Abstract: Fusion systems made their first appearance in a 2006 paper by Puig and have since then been investigated by many researchers around the world. A fusion system is a structure that encodes the properties of conjugation between p-subgroups of a group, for p any prime number.
Given a finite group G, it is always possible to define the saturated fusion system realized by G on one of its Sylow p-subgroups S: this is the category where the objects are the subgroups of S and the morphisms are the restrictions of conjugation maps induced by the elements of G.
However, not all saturated fusion systems can be realized in this way:
when this is the case, we say that the fusion system is exotic. An important research direction involves the study of the behavior of exotic fusion systems (in particular at odd primes).

In this talk we will present an overview of recent results concerning the classification of saturated fusion systems on certain families of finite p-groups, highlighting the developments on the understanding of exotic fusion systems at odd primes.

Josh Maglione

Title: *Automorphisms of groups from elliptic curves*

Abstract: We describe a group scheme coming from an elliptic curve over a field K. We characterize when two (abstract) groups arising in this way are isomorphic, and we use this to describe the automorphism group of such groups.
This generalizes work of du Sautoy--Vaughan-Lee and Stanojkovski--Voll.
This is joint work with Mima Stanojkovski.

Gunter Malle

Title: *Brauer's height zero conjecture*

Abstract: Brauer's height zero conjecture relates heights of characters in a Brauer block of a finite group to the structure of its defect groups. We report on the history and on the recent solution of this conjecture. This is joint work with Gabriel Navarro, Mandi Schaeffer Fry and Pham Huu Tiep, and building on previous work of many authors.

Kevin Piterman

Title: *Advances on Quillen's conjecture*

Abstract: The study of the p-subgroup complexes began motivated by group cohomology and equivariant cohomology of topological spaces "modulo the prime p". For example, Kenneth Brown proved that the reduced Euler characteristic of this complex is divisible by the size of a Sylow p-subgroup, giving rise to a sort of "Homological Sylow theorem". Later, he showed that the mod-p equivariant cohomology of the p-subgroup complex of a finite group coincides with the mod-p cohomology of the group. Deeper relations with finite group theory, representation theory, and finite geometries were also explored. For instance, uniqueness of certain simple groups, finite geometries for sporadic groups, Lefschetz modules, and, more recently, endotrivial modules.

In 1978, Daniel Quillen conjectured that the poset of non-trivial p-subgroups of a finite group G is contractible if and only if G has non-trivial p-core. Quillen established the conjecture for solvable groups and some families of groups of Lie type. The major step towards the resolution of the conjecture was done by Michael Aschbacher and Stephen D. Smith at the beginning of the nineties. They roughly proved that if p>5 and G is a group of minimal order failing the conjecture, then G contains a simple component PSU(n,q^2) failing a certain homological condition.

In this talk, I will present new advances in the conjecture, with a focus on the prime p=2, which was not covered by the methods developed by Aschbacher-Smith. For example, we will see that sporadic groups cannot appear as components in a minimal counterexample to the conjecture for odd primes p, and state a slightly more restrictive result for p=2. In particular, we can conclude that sporadic and alternating components are (roughly) not an obstruction to establishing Quillen's conjecture for any prime p. This is joint work with S.D. Smith.

Sarah Rees

Title: *Relating Artin groups to their monoids*

Abstract: I'll talk about Artin groups and their monoids.

Artin group are defined by their finite presentations; the class of all Artin groups is very general, containing groups with (apparently) quite a range of properties. Some of these groups (e.g. the braid groups) have quite natural geometric origins, but for most of them, very little geometric information is known. There are a number of problems that are solved for particular classes of Artin groups but not in general. Among these are the word problem and the K(π, 1)-conjecture (of the contractibility of the group's Deligne complex), both of those solved decades ago for braid groups.

The last decade has shown quite significant progress in the study of geodesic words and of the word problem in a variety of Artin groups, and work of Dehornoy suggests that words over the group generators can be effectively studied as *multifractions*, that is, as words over the the Artin monoid. Meanwhile, work of Boyd suggests study of the groups's Deligne complex via the Deligne complex for the monoid.

I shall discuss progress in rewriting in Artin groups (referring to my own work with Holt, work of Blasco, Cumplido and Morris-Wright, and to Dehornoy's work), and explain how this could be used in an investigation of the Deligne complex,in my very recent work with Boyd, Charney and Morris-Wright.

Gerhard Röhrle
Title: *G-complete reducibility and semisimplification*
Abstract: Let G be a reductive algebraic group---possibly non-connected---over a field k and let H be a subgroup of G. If $G= GL_n$ then there is a degeneration process for obtaining from H a completely reducible subgroup H' of G; one takes a limit of H along a cocharacter of G in an appropriate sense. We generalise this idea to arbitrary reductive G using the notion of G-complete reducibility and results from geometric invariant theory over non-algebraically closed fields. Our construction produces a G-completely reducible subgroup H' of G, unique up to $G(k)$-conjugacy, which we call a k-semisimplification of H. This gives a single unifying construction which extends various special cases in the literature (in particular, it agrees with the usual notion for $G= GL_n$ and with Serre's “G-analogue" of semisimplification for subgroups of $G(k)$).
There is an analogue of the notion of G-complete reducibility over k for a Lie subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g} = Lie(G)$ of G and an analogous concept of a k-semisimplification $\mathfrak{h}'$ of $\mathfrak{h}$; $\mathfrak{h}'$ is a Lie subalgebra of $\mathfrak{g}$ associated to $\mathfrak{h}$ which is G-completely reducible over k. This is the Lie algebra counterpart of the analogous notion for subgroups from above. As in the subgroup case, we show that $\mathfrak{h}'$ is unique up to $Ad(G(k))$-conjugacy in $\mathfrak{g}$. Moreover, we prove that the two concepts are compatible: for H a closed subgroup of G and H' a k-semisimplification of H, the Lie algebra $Lie(H')$ is a k-semisimplification of $Lie(H)$.
This is a report on joint work with Sören Böhm, Michael Bate, Benjamin Martin and Laura Voggesberger.

Colva Roney-Dougal
Title: *Counting permutation groups*
Abstract: One of the most elementary, but difficult, questions we can ask about a finite group is how many subgroups it has. For the symmetric group on n points, an elementary argument easily gives a rather large lower bound on the number of subgroups, and it was conjectured by Pyber in 1993 that up to lower order error terms this is also an upper bound. This talk will present an answer to Pyber's conjecture. This is joint work with Gareth Tracey.

Wilberd van der Kallen
Title: *Reductivity and finite generation*
Abstract: Recall that the First Fundamental Theorem of Invariant Theory is about finite generation of the subalgebra of invariants, when a reductive group acts on an algebra. Nowadays one knows such a result even after replacing the base field with an arbitrary commutative noetherian ring. We review some of the history.

Richard Weiss
Title: *Tits polygons*
Abstract: Tits polygons are generalizations of Moufang polygons. Most Moufang polygons arise as the spherical buildings that correspond to absolutely simple algebraic groups of rank 2. All Moufang polygons can be coordinatized by algebraic data that is required, in each case, to be anisotropic in an appropriate sense. There are Tits polygons that can be coordinatized by the same algebraic data, but in the isotropic case. The best known examples are the “projective planes” over the octonions. These are, in fact, Tits triangles coordinatized by split octonion algebras, i.e. octonion algebras whose norm is isotropic. These triangles have a simple description in terms of buildings of type E6. In fact, every irreducible spherical building of rank at least 3 gives rise to Tits polygons—sometimes more than one—in a similar way. In this talk we will describe classification results for Tits polygons and applications of Tits polygons to the study of Moufang sets. (Moufang sets are, roughly speaking, the 2-transitive permutation groups that have a split BN -pair of rank 1.) This is joint work with Bernhard Mühlherr.

Stefan Witzel
Title: *Infinite groups and coarse geometry*
Abstract: Pretending (albeit not rightfully) finite group theory to be "done", infinite groups move into focus. In order to avoid redoing finite group theory, infinite groups are studied up to a notion of equivalence that at least includes commensurability. One the most widely considered notions is that of coarse equivalence.
I will talk about some invariants under coarse equivalence and how they relate to classical notions.

Claus-Michael Ringel: *Bernd Fischer at Bielefeld - some recollections*

Gina Keiko Friesicke:
Auszüge aus Bach, Solosonate No 3
Ysaye, Solosonate No 4
Paganini, Caprice No 4